

**Sustainable Development of Mineral Resources**

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**Abstract:** Human Civilization heavily depends on optimal mining of non renewable but finite mineral resources and winning technology for valuable metals and trace elements such as Cu, Zn, Fe, Al, Cr, Ni, Co and atomic elements such as U, Th. **In past, unscientific mining and beneficiation of minerals have depleted high grade ores, Hence we need to mine ores at greater depths now and of low grades at much higher cost. So sustainable mining urgently requires in minimization of cost of products and maximization of net profits. Mathematical statistics is the technology of 21st century for optimal and sustainable growth.** Representative elementary volume (REV) is adequate sample size to obtain stable statistics for mineral deposits. For single population, statistical tests such as Hypothesis test, Regression and correlations, principal components and factors, multiple regression partial and canonical correlations are useful for scalar random variables. Similarly, for multiple population, hypothesis tests include ANOVA and ANCOVA, MANCOVA, LDF, QDF and are useful for vector random variables. **Sustainable development of mineral resources includes extension of mine life (Reserve/yearly extraction rate) to its maximum value through minimization of extraction rate.** However, this extraction rate is dynamic and must be periodically updated to its current value based on available reserve and market price of the minerals or metals.

**Keywords:** Sustainable Development, Mineral Resources, Mathematical statistics,

**Introduction**

Rocks and ores comprise multi-component hierarchical minerals, molecules & elements/isotopes/ions. Man has successfully exploited these valuable constituents in mineral resources for economic, industrial and social growth since beginning of our civilization and thereby creating severe depletion of high-grade ores at Earth's surface and shallow depths as well as of damage to ecology, environment and human health. Hence, sustainable growth of mineral industries urgently requires intensive exploration of low-grade and deeper mineralizations, which imply optimal and very efficient mining efforts based on spatial and/or spatio-temporal distribution of these constituents. Optimal and efficient mining insure maximization of profits with associated waste disposal with minimum damage to ecology-environment.

Mineral resources include mineable ores, low-grade ores mineable with blending high-grade ores and/or using suitable beneficiation techniques, or lean ores left in-situ for the future when profits are possible with increased ore prices. Wastes are intimately mixed with ore minerals and can be separated by proper grinding to fine sizes, blending, beneficiation, and floatation techniques. These wastes must be disposed off by the mining company without damage to ecology and environment (required by Law).

Mineral resources, including energy resources as coal, oil and gas, and radioactive elements, are formed by very long periods of geological time and hence are necessarily finite, non-renewable and exhaustible. Therefore, their exploitation must be performed with much care and efficiency with suitable optimization techniques at every stage of mining activities to insure sustainable national economy and societal benefits. Statistical techniques are most useful for optimal mining by proper delineation of ore and waste blocks, mine planning, identifying pathfinders for ore exploration and discrimination of high-, medium-, low- grade ores from waste blocks.

Adequate sample volume/weight (Representative elementary volume, REV) are collected from mineralizations to characterize their fractional mineral, and elemental concentrations ( $x$ , with  $0 < x < 1$ ) and a large number of samples ( $> 20$ ) insure proper statistical analysis. Major ( $> 10\%$ ), Minor ( $1\% < x\% < 10\%$ ), and Trace components ( $< 1\%$ ) are present within samples and vary randomly (rvs) among samples and their probability density function (PDF) is usually not Gaussian/Normal for statistical analyses, hypotheses tests and optimal decisions. Hence, prior nonlinear transformation of fractional concentrations of elements is a must for statistical estimations and analyses. The fractional proportions are considered continuous random variables and a  $\log(x/(1-x))$  transform Gaussianize/Normalize this rvs for further statistical analysis. For trace components,  $x$  tends to zero and  $(1-x)$  tends to 1, and therefore the above log transform reduces to a much simpler  $\log(x)$  transform which is well-known as log-normal distribution of trace elements in geochemistry. Rocks and ores contain 5/6 minerals and 10/12 major elements/ions with several trace elements/ions which are thermodynamically at equilibrium in closed systems with two degrees of freedom s.a. Pressure(P) and temperature(T). However, in Open Systems s.a. hydrothermal/metamorphic rocks/ores the degrees of freedom is larger depending on number of existing mineral phases. Major problems for estimation of fractional proportions of minerals/elements are following:

- (a) Constant Sum or Closure: since total volume/weight of analysed sample is a fixed constant (1.0 or 100%), major and minor constituents are not independent rvs, which induce spurious negative correlations for these constituents within samples, thereby jeopardizing statistical inferences,
- (b) The data matrix is neither full-rank ( $C =$  number of constituent rvs) nor has unique inverse for later statistical analyses but has a lower rank ( $C - 1$ ) with a non-unique inverse.

Mineral resources are National or State properties which are leased for a fixed period of time to either government or private parties for mining and marketing of ores. Heavy penalties must be imposed if any mining laws are broken to insure public health, sustainable growth, and no damage to ecology. Rapid extraction of high-grade ores without proper blending and/or beneficiation of low-grade ores, induce lower mine-life and poor sustainability. Optimal decisions are absolutely necessary at all stage of mining and marketing of ores, summarized below:

- a. Mining Stage: Optimal method of mining; Mine plans in 3D based on as low a cut-off grade; Dynamic extraction rates based on ratio of sale to cost prices; horizontal and vertical extensions.
2. Blending and beneficiation: Optimal grinding size; conservation high- and medium-grade ores.
3. Waste disposal: Optimal transport and safe disposal of wastes insuring no ecological damage.
4. Marketing: Optimal classes of ores by blending/ beneficiation, use of low grade ores left insitu / in the mine dumps.
5. Mine closure: Planation of damaged Earth's surface; afforestation of the mine areas.

Hence, optimal mining of non-renewable mineral resources involve highly complex non-linear dynamic processes. These processes are to be linearized and simplified to insure optimal decisions to maximize national economic growth with sustainability improve social benefits and preserve ecology over the life-span of the mine.

### **Exploration and Mining**

Population growth and ever-increasing demands for greater living standards of citizens have induced rapid exploitation of non-renewable mineral and energy resources at surface and shallow depths, implying critical levels of deterioration of ecology with high health hazards and risks. Therefore, we urgently need intensive exploration efforts for concealed and deeper mineralization at much higher costs and increased efficiency/technology.

Geochemical exploration is cheapest quantitative detection technique for such hidden targets both at local as well as regional scales. Integration of all relevant information from remotely sensed data, geology, geochemistry and geophysics can help to delineate homogeneous parts of the crust for detecting mineralization which has to be confirmed by optimized drilling and sampling. Then, proved, probable and possible reserves and associated grades can be estimated for mine feasibility, mine planning and to decide on blending and beneficiation techniques to be adopted.

At the exploration stage, data are sparse and geochemical anomalies can be detected directly from the valuable elements distribution in space or using a proxy pathfinder elemental distribution. However, at mine development and mine production stages data become sufficiently large to apply more reliable statistical analyses (using Multivariate Normal and Time/Spatial series) to estimate regional/local background values and delineate positive anomalies for ore targeting and negative anomalies for disposal of toxic wastes.

Since fractional proportions ( $0 < x < 1$ ) of elements/ions/isotopes in ores are not distributed by Gaussian Law, simple one-to-one pre-transform such as  $\log(x/(1-x))$  can Gaussianize/Normalize the distributions. But trace components ( $x$ ) tend to zero inducing  $(1-x)$  tending to 1, and this reduces the above log-transform to simpler  $\log(x)$ ; well-known log-Normal distribution of trace elements in Geochemistry. Population parameters (mean and variance/ its positive square root called standard deviation) are then estimated from sample statistics with greater accuracy and precision as sample size ( $N$ ) increases. Two important hypotheses tests are: (I). Ho/ Null Hypothesis: of sample mean (average  $x$ ) equal to given population mean ( $\mu$ ) with Alternative Hypothesis  $H_1$  (average  $x$ ) NOT equal to Population mean; (II). Homogeneity of Sample variances (Ho: all sample variances are equal) versus at least one sample variance is different from the rest ( $H_1$ ). Decisions involve two types of Errors(called Type I and Type II errors): Type I Error is rejecting Ho when it is True; Type II Error is accepting  $H_1$  when it is False. It is prudent to maximize  $(1 - \text{Type II error})$ , the Power of Hypothesis Test for any given Type I error.

### **Some Applications of Statistics in Mining**

Statistics is the technology 21<sup>st</sup> Century and along with the current capabilities of computers, this technology will be essential, and highly beneficial for mining and mineral processing industries. A  $\log(x/(1-x))$  pre-transform of fractional proportions of constituents provides the desired independence of transformed rvs in spatially distance samples and a Gaussian/Normal PDF for each constituent. Then parameters for each constituent can be estimated in large ( $N > 20$ ) samples and necessary Hypotheses tests and optimal decisions can be taken using UNIVARIATE, MULTIVARIATE and/ TIME (SPATIAL) SERIES METHODS as required. Optimal decisions induce Profit Maximization and Minimum Environmental Damage with higher sustainability, economic and societal growths.

### **Identification of Pathfinders**

Pathfinders are most useful in geochemical exploration as these can be analysed cheaply and occur everywhere in ores. Correlation matrix ( $R$ ) of pre-transformed values is computed using R-mode (order of matrix is  $C \ll N$ ), where absolute values of correlation coefficient (abs value of  $r$ ) is STRONG if  $> 0.50$  and medium if  $0.30 < \text{abs } r < 0.50$  and can be neglected if  $< 0.30$ . Factor analysis provide rotated factors that are strongly loaded (absolutely correlated) with a few constituents yielding pathfinders(proxies). Multi-element or Multi-mineral ores would

require CANONICAL CORRELATION analyses with mineable constituent(s) as CRITERION and the other sets as PREDICTORS/ CONTROL vectors.

### **Mine Feasibility**

Sustainable mining must insure that expected profits per year remains positive and substantial to meet cash flows and financial commitments. Thus high sale value of high-grade ores and/or beneficiated low-grade ores in market is essential (relative to cost of mining operations). This is a complex non-linear problem which cannot be linearized. Smaller mines with less low-grade ores must pool their non-marketable resources and beneficiate these in a single cooperatively operated plant with proportional cost and profit sharing.

### **3D Modelling and Mine Planning**

Fast, efficient and upto date computer system having links to each end-user is essential for this purpose.

1. Fast transmission of databases, maps and sections to central processor online/offline
2. preparation of 3D maps using GIS technology, geological sections for mine
3. planning/operations
4. Optimal plans for transport, blending and beneficiation; timeframe for all these works
5. Marketing plans, expected vs. actual profits, asset doubling in 5/6 years
6. Monitoring ecological damage and plan mitigation proc
7. Mine closure plan, disposal of equipment and personnel, corporate social responsibility(CSR)

### **Mine Sustainability**

1. Market incentives with pollution control focusing on management system(MS)
2. Mine Closure with EIA and SIA at all stages (E = Environment; S = Social)
3. Bonds may be issued to clean-up pollution after mine closure
4. Obtain environmental and social performance indicators, Risk assessment for envt. Management(EM), life cycle assessments(LCA), Technology Choice and EMS choice

### **R & D efforts for Resource Augmentation**

1. Exploration of New ore deposits
2. Invention of natural and/or technological substitutes
3. Waste treatment and safe disposal
4. Optimal marketing of lean ores by blending with local/ imported high-grade ores
5. Optimal grinded size for liberation of ore minerals and beneficiation; optimal mix of products
6. Optimal cutoff grade estimation to increase sustainable mining and mine life
7. Profit maximization by dynamic optimization performed periodically within Govt. policy and existing mining, environmental, and forest Laws

### **Conclusion**

i. Fractional constituents ( $x$ ; random variable,  $rv$ ) in rocks/ores form multicomponent system with less than full-rank for statistical analyses and inferences. A  $\log(x/(1-x))$  pre-transform eliminate rank problem and simultaneously insures the distribution becomes Gaussian/Normal for linear statistical analyses using univariate and/or multivariate models. However, if samples are closely spaced, these  $rvs$  are correlated and time/spatial series analyses become appropriate.

ii. Geochemical fractional concentrations are averaged mean values on REV samples over 3D space (volume/weight) and are spuriously negatively correlated within sample because of closure and possess non-constant variances over mean values. Hence,  $\log(x/(1-x))$  transform eliminates this spurious negative correlations and makes the distribution linear and Gaussian/Normal for parameter estimations, hypotheses tests, and other statistical/geological inferences using UNIVARIATE/MULTIVARIATE statistics.

iii. Mineralisations involve highly complex nonlinear geological processes, needing nonlinear methods such as nearest neighbors(NN), fuzzy logic (FL), genetic algorithm(GA) and soft computing (SC) techniques to solve these problems.

iv. During exploration and development stages when samples are not very large, multivariate methods such as Factor Analysis identify pathfinders (proxy to ore mineral/element); two-group and multi-group linear discriminant functions to delineate high-grade, medium-grade, low-grade ores from waste blocks. However, at the production stage, it is prudent to identify different categories of ore zones such as: Marketable grade; Marketable after blending; Marketable after beneficiation, Low-grade ores left insitu for future; and Waste materials left insitu/or mined to be disposed off. A multigroup discriminant function approach can be used to delineate these categories.

v. Optimal use of unmarketable ores by blending/beneficiation accumulates additional profits, reduce waste disposal and pollution and increase sustainability of mining industry.

vi. Cutoff grade in mining is most important decision since it insures maximum profits and increases sustainability. However, it involves several complex geological, spatial assay distributional and many economic factors such as sales ( $s$ ), cost ( $c$ ) prices of marketable ores, and also costs of waste treatment /disposals. Static problem for cutoff grade estimation has been solved by author; but dynamic problem needs solution at different time/time-interval of actual mine operation.

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